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EFFECT OF MAGNETIC FIELD ON THERMOSOLUTAL CONVECTION IN A ROTATING NON-NEWTONIAN NANOFLUID WITH POROUS MEDIUM

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ABSTRACT

The current study examines the influence of rotation and magnetic field on Rivlin-Ericksen nanofluid permeated with a porous media, which is heated from below. This nanofluid model takes into account thermophoresis and Brownian motion phenomena. The momentum-balance equation is stimulated under the influence of nanoparticles, viscoelasticity, rotation and magnetic field. Linear stability theory is used to identify the prerequisite for the onset of convection. The effect of thermo-nanofluid Lewis number, Taylor number, modified diffusivity ratio, medium porosity, nanoparticle Rayleigh number, solutal Rayleigh number, thermo-solutal Lewis number, Soret and Dufour parameters have been examined analytically and graphically. It is observed that the thermal nanofluid Lewis number, Dufour parameter, Chandrasekhar number, Taylor number, modified diffusivity ratio, solutal Rayleigh number, medium porosity, thermo-solutal Lewis number and Soret parameter have a strengthening influence on steady-state convection whereas nanoparticles Rayleigh number have let down influence on steady-state convection.

KEYWORDS: *Nanofluid, Porous Medium, Thermosolutal Convection, Magnetic Field, Rotation*

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1. INTRODUCTION

Thermosolutal convection is an important problem, which finds applications in astronomy, geophysics, limnology, food processing, engineering and modeling oil reservoirs, etc. There are many diffusive elements in nature. Veronis[1] investigated the fluid that is heated and soluted from below. Eastman et al.[2] first used the phrase “nanofluid” which describes a combination of nanoparticles with a regular base fluid. Buongiorno [3] analyzed the problem in nanofluids convective transport. He carried forward Choi work. Rana and Sharma [4] addressed the problem of rotation and magnetic field effects on thermolsolutal convection in a compressible Walters' (Model B') infiltrated by suspended particles in a porous medium, where the particles are suspended while rotating and concluded that the rotation stabilizes the system while the suspended particles destabilize the system.

A layer of nanofluid in a porous media was investigated by Rana et al. [5] who concluded that the Walters' (Model B') nanofluid behaved like a typical Newtonian nanofluid. Rana and Jamwal [6] thermal instabilities of compressible Walters' (Model B') In hydromagnetics, a rotating fluid permeated with suspended particles (fine dust) in a porous medium is considered. The magnetic field has a destabilizing effect in the absence of rotation and a stabilizing or destabilizing effect in the presence of rotation under certain conditions. Different authors studied [7-9] analyzed the double-diffusive convection and concluded that both rotation and magnetic field stabilize the system. Sharma et al. [10-12] investigated the influence of rotation and magnetic field on thermosolutal convection in viscoelastic nanofluid with porous medium and it was discovered that the Dufour parameter, solutal Rayleigh number and thermosolutal Lewis number are destabilizing the system towards stationary modes.

The quick review of prior work makes it clear that investigations involving variables like rotation, magnetic field and nanofluid have not been carried out compositely on Rivlin-Ericksen. Therefore, the current investigation aims to determine how rotation and magnetic field affect the given problem.

2. MATHEMATICAL MODEL

Consider a rotating layer of Rivlin-Ericksen nanofluid of width $d$with velocity (angular) Ω and is subjected to magnetic field $\boldsymbol{h} = (0, 0, h)$positioned between the plates $z = 0$ and$z = 0$under the gravitational force $\boldsymbol{g} = (0, 0, -g)$ as shown in Figure. The fluid layer is heated from below and working upwards direction. The upper boundary and lower boundary are taken to be T_1, C_1, φ_1andT_0, C_0, φ_0 respectively, with $T_0 > T_1, C_0 > C_1$ and $\varphi_0 > \varphi_1$.

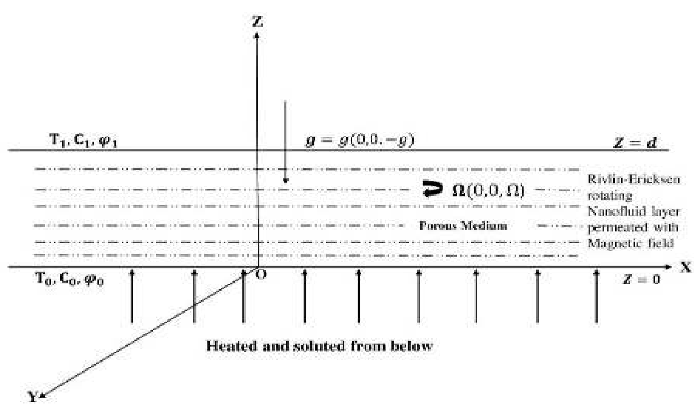

Figure 1: Physical Configuration.

3. GOVERNING EQUATIONS

The governing equations for Rivlin-Ericksen nanofluid in the porous medium as given by Sharma et al. [10] Chandrasekhar[13], Kuznetsov and Nield[14] and Nield and Kuznetsov [15] are:

$$\nabla . \boldsymbol{q} = 0, \quad (1)$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}.\nabla)\right]\boldsymbol{q} = -\nabla p+\rho\boldsymbol{g}-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\boldsymbol{q}+\frac{\mu_e}{4\pi}(\boldsymbol{h}.\nabla)\boldsymbol{h}+\frac{2\rho}{\varepsilon}(\boldsymbol{q}\times\Omega). \tag{2}$$

The density of nanofluid can be written as

$$\rho = \varphi\rho_p + (1-\varphi)\rho_f. \tag{3}$$

We approximate the density of the nanofluid by that of the base fluid, that is, we consider $\rho = \rho_f$.

Now, introducing the Boussinesq approximation for the base fluid, the specific weight, $\rho\boldsymbol{g}$ in equation (2) becomes

$$\rho\boldsymbol{g} \approx (\varphi\,\rho_p + (1-\varphi)\,\{\rho\,(1-\alpha_T\,(T-T_0)-\alpha_c\,(C-C_0))\})\,\boldsymbol{g}. \tag{4}$$

If one introduces a buoyancy force, the equations of motion for Rivlin- Ericksen nanofluid by using Boussinesq approximation and Darcy model for porous medium is given by

$$\frac{\rho}{\varepsilon}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla p+\left(\varphi\rho_p+(1-\varphi)\{\rho\left(1-\alpha_T(T-T_0)-\alpha_c(C-C_0)\right)\}\right)\boldsymbol{g}-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\boldsymbol{q}+\frac{\mu_e}{4\pi}(\boldsymbol{h}.\nabla)\boldsymbol{h}+\frac{2\rho}{\varepsilon}(\boldsymbol{q}\times\Omega). \tag{5}$$

For nanoparticles, the continuity equation given by is

$$\frac{\partial\varphi}{\partial t}+\boldsymbol{q}.\nabla\varphi = D_B\nabla^2\varphi+\frac{D_T}{T_1}\nabla^2 T. \tag{6}$$

For the nanofluid, the equation of thermal energy is given by

$$(\rho c)_m\frac{\partial T}{\partial t}+(\rho c)_f\boldsymbol{q}.\nabla T = k_m\nabla^2 T+\varepsilon(\rho c)_P\left(D_B\nabla\varphi\nabla T+\frac{D_T}{T_1}\nabla T.\nabla T\right)+(\rho c)_f D_{TC}\nabla^2 C. \tag{7}$$

The equation of conservation of solute concentration is given by

$$\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}\boldsymbol{q}.\nabla C = D_{SM}\nabla^2 C+D_{CT}\nabla^2 T. \tag{8}$$

The Maxwell equations are given as

$$\frac{\partial \boldsymbol{h}}{\partial t}+(q.\nabla)\boldsymbol{h} = (\boldsymbol{h}\nabla)\boldsymbol{q}+\eta\nabla^2\boldsymbol{h}, \tag{9}$$

$$\nabla.\boldsymbol{h} = 0. \tag{10}$$

The boundary conditions are given by

$$\left.\begin{array}{l} w = 0, T = T_0, \varphi = \varphi_0, C = C_0 \text{at} z = 0 \\ w = 0, T = T_1, \varphi = \varphi_1, C = C_1 \text{at } z = d \end{array}\right\}. \tag{11}$$

We establish nondimensional variables as

$$(x^*,y^*,z^*) = \frac{(x,y,z)}{d}, \boldsymbol{q}^* = \boldsymbol{q}\frac{d}{\kappa_m}, t^* = \frac{t\kappa_m}{\sigma d^2}, p^* = \frac{pk_1}{\mu\kappa_m}, \varphi^* = \frac{\varphi-\varphi_0}{\varphi_1-\varphi_0}, \quad T^* = \frac{T-T_1}{T_0-T_1}, C^* = \frac{C-C_1}{C_0-C_1}, \boldsymbol{h}^* = \frac{\boldsymbol{h}}{h_0}$$

where, $\kappa_m = \frac{k_m}{(\rho c)_f}, \sigma = \frac{(\rho c)_m}{(\rho c)_f}$ are thermal diffusivity of the fluid and the thermal capacity ratio respectively. Dropping the star ($*$) for simplification. Equation (1) and equations (5) to (10) in non-dimensional form reduces to

$$\nabla.\boldsymbol{q} = 0, \tag{12}$$

$$\frac{1}{\sigma V_a}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla p - \left(1 + F\frac{\partial}{\partial t}\right)\boldsymbol{q} - R_m\hat{k} - R_n\varphi\hat{k} + R_aT\hat{k} + \frac{R_S}{L_e}C\hat{k} + Q\frac{Pr_1}{Pr_2}(\boldsymbol{h}.\nabla)\boldsymbol{h} + \sqrt{T_a}(\boldsymbol{q}\times\hat{k}), \tag{13}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t} + \boldsymbol{q}.\nabla\varphi = \frac{1}{L_n}\nabla^2\varphi + \frac{N_A}{L_n}\nabla^2T, \tag{14}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{q}.\nabla T = \nabla^2T + \frac{N_B}{L_n}\nabla\varphi.\nabla T + \frac{N_AN_B}{L_n}\nabla T.\nabla T + N_{TC}\nabla^2C, \tag{15}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\boldsymbol{q}.\nabla C = \frac{1}{L_e}\nabla^2C + N_{CT}\nabla^2T, \tag{16}$$

$$\frac{\partial\boldsymbol{h}}{\partial t} + \sigma(\boldsymbol{q}.\nabla)\boldsymbol{h} = \sigma(\boldsymbol{h}\nabla)\boldsymbol{q} + \sigma\frac{Pr_1}{pr_2}\nabla^2\boldsymbol{h}, \tag{17}$$

$$\nabla.\boldsymbol{h} = 0. \tag{18}$$

where the dimensionless parameters are

thermosolutal Lewis number $L_e = \frac{\kappa_m}{D_{SM}}$, thermo-nanofluid Lewis number $L_n = \frac{\kappa_m}{D_B}$, kinematic viscoelastic parameter F = $\frac{\mu'\kappa_m}{\mu\sigma d^2}$, density Rayleigh number $R_m = \frac{(\rho_P\varphi_0+\rho(1-\varphi_0))gk_1d}{\mu\kappa_m}$, nanoparticle Rayleigh number $R_N = \frac{(\rho_P-\rho)(\varphi_1-\varphi_0)gk_1d}{\mu\kappa_m}$, Darcy thermal Rayleigh number $R_a = \frac{\rho\alpha_T(T_0-T_1)gk_1d}{\mu\kappa_m}$, solutal Rayleigh number $R_S = \frac{\rho\alpha_C(C_0-C_1)gk_1d}{\mu D_{sm}}$, Prandtl number $Pr_1 = \frac{\mu}{\rho\kappa_m}$, magnetic Prandtl number $Pr_2 = \frac{\mu}{\rho\eta}$, Chandrasekhar number $Q = \frac{\mu_e h_0^2 k_1}{4\pi\eta\mu}$, modified diffusivity ratio $N_A = \frac{D_T(T_0-T_1)}{D_BT_1(\varphi_1-\varphi_0)}$, modified particle density increment $N_B = \frac{\varepsilon(\rho c)_p(\varphi_1-\varphi_0)}{(\rho c)_f}$, Dufour parameter $N_{TC} = \frac{D_{TC}(C_0-C_1)}{\kappa_m(T_0-T_1)}$, Soretparameter $N_{CT} = \frac{D_{TC}(T_0-T_1)}{\kappa_m(C_0-C_1)}$, Taylor number $T_a = \left(\frac{2\Omega d^2\rho}{\mu}\right)^2$, Vadasz number $V_a = \frac{\varepsilon Pr_1}{P_l}$, medium Permeability $P_l = \frac{k_1}{d^2}$.

The dimensionless boundary conditions are

$$\left.\begin{array}{l} w = 0, T = T_0, \varphi = \varphi_0, C = C_0 \text{ at } z = 0 \\ w = 0, T = T_1, \varphi = \varphi_1, C = C_1 \text{ at } z = 1 \end{array}\right\}. \tag{19}$$

4. BASIC STATE SOLUTIONS

Following Kuznetsov and Nield[14]Nield and Kuznetsov [15] and Sheu[16]. The basic state of nanofluid does not depend on time and is described as

$$\boldsymbol{q}(u,v,w) = 0, p = p_b(z), C = C_b(z), T = T_b(z), \varphi = \varphi_b(z), \boldsymbol{h} = (0,0,1). \tag{20}$$

Using these basic state solutions defined by equation (20), equations (12) - (18) reduce to

$$0 = -\frac{d}{dz}p_b(z) - R_m - R_n\varphi_b(z) + R_aT_b(z) + \frac{R_S}{L_e}C_b(z), \tag{21}$$

$$\frac{d^2}{dz^2}\varphi_b(z) + N_A\frac{d^2}{dz^2}T_b(z) = 0, \tag{22}$$

$$\frac{d^2}{dz^2}T_b(z)+\frac{N_A}{L_n}\frac{d}{dz}\varphi_b(z)\frac{d}{dz}T_b(z)+\frac{N_A N_B}{L_n}\left(\frac{d}{dz}T_b(z)\right)^2+N_{TC}\frac{d^2}{dz^2}C_b(z)=0, \quad (23)$$

$$\frac{1}{L_e}\frac{d^2}{dz^2}C_b(z)+\ N_{CT}\frac{d^2}{dz^2}T_b(z)=0. \quad (24)$$

According to Buongiorno[3], for most nanofluid investigated so far $\frac{L_n}{(\varphi_1-\varphi_0)}$ is large, of order $10^5-10^6$and since the nanoparticle fraction decrement $(\varphi_1-\varphi_0)$ is not smaller than 10^{-3} which means L_n is large.

$$T_b=1-z, C_b=1-z\text{and}\varphi_b=z. \quad (25)$$

5. PERTURBATION SOLUTIONS

We introduce small perturbation on the basic state for investigating the stability of the system as

$$\boldsymbol{q}(u,v,w)=0+\boldsymbol{q}'(u,v,w), T=(1-z)+T', C=(1-z)+C', \varphi=z+\ \varphi', p=p_b+p', \boldsymbol{h}=(0,0,1)+\boldsymbol{h}'. \quad (26)$$

Using equation (26) into equations (12) to (18) and linearizing the resulting equations by neglecting nonlinear terms that are product of perturbations (dropping the primes $(')$), the following equations are obtained

$$\nabla.\boldsymbol{q}=0, \quad (27)$$

$$\frac{1}{\sigma V_a}\frac{\partial \boldsymbol{q}}{\partial t}=-\nabla p-\left(1+F\frac{\partial}{\partial t}\right)\boldsymbol{q}-R_N\varphi\hat{k}+R_aT\hat{k}+\frac{R_S}{L_e}C\hat{k}+Q\frac{Pr_1}{Pr_2}\frac{\partial \boldsymbol{h}}{\partial z}\hat{k}+\sqrt{T_a}(\boldsymbol{q}\times\hat{k}), \quad (28)$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t}+\frac{1}{\varepsilon}\mathrm{w}=\frac{1}{L_n}\nabla^2\varphi+\frac{N_A}{L_n}\nabla^2T, \quad (29)$$

$$\frac{\partial T}{\partial t}-w=\nabla^2T+\frac{N_B}{L_n}\left(\frac{\partial T}{\partial z}-\frac{\partial\varphi}{\partial z}\right)-2\frac{N_AN_B}{L_n}\frac{\partial T}{\partial z}+N_{TC}\nabla^2C, \quad (30)$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}-\frac{1}{\varepsilon}w=\frac{1}{L_e}\nabla^2C+N_{CT}\nabla^2T, \quad (31)$$

$$\frac{\partial\boldsymbol{h}}{\partial t}=\sigma\frac{\partial w}{\partial z}\hat{k}+\sigma\frac{Pr_1}{pr_2}\nabla^2\boldsymbol{h}, \quad (32)$$

$$\nabla.\boldsymbol{h}=0. \quad (33)$$

The boundary conditions after perturbation becomes

$$w=0, T=0, \varphi=0, C=0 \text{ at } z=0 \text{ and } z=1. \quad (34)$$

Operating equation (28) with $\hat{k}\,.curl.curl$, $(i.e.$Making use of result $curl.curl\ =\ grad.div\ -\ \nabla^2)$ we get

$$\frac{1}{\sigma V_a}\frac{\partial}{\partial t}\nabla^2w+\left(1+F\frac{\partial}{\partial t}\right)\nabla^2w=R_a\nabla_H^2T-R_N\nabla_H^2\varphi+\frac{R_S}{L_e}\nabla_H^2C+Q\frac{\partial^2w}{\partial z^2}+T_a\frac{\partial^2w}{\partial z^2}, \quad (35)$$

where, $\nabla_H^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator on the horizontal plane.

6. NORMAL MODE ANALYSIS

The disturbances analysed by normal mode analysis as follows

$$[w, T, C, \varphi] = [W(z), \Theta(z), \Gamma(z), \Phi(z)] \exp\,(ik_x x + ik_y y + nt), \tag{36}$$

where n is the growth rate and k_x and k_y are the wave number along x and y directions, respectively.

Using equation (36) in equations (29), (30), (31) and (35), we get

$$\left[\left(1 + nF + \frac{n}{\sigma V_a}\right)(D^2 - a^2) + QD^2 + T_a D^2\right] W + R_a a^2\, \Theta + \frac{R_S}{L_e} a^2 \Gamma - R_N a^2\, \Phi = 0, \tag{37}$$

$$\frac{1}{\varepsilon}\mathrm{W} - \frac{N_A}{L_n}(D^2 - a^2)\Theta + \left[\frac{n}{\sigma} - \frac{(D^2 - a^2)}{L_n}\right]\Phi = 0, \tag{38}$$

$$W + \left[(D^2 - a^2) + \frac{N_B}{L_n}D - 2\frac{N_A N_B}{L_n}D - n\right]\Theta + N_{TC}(D^2 - a^2)\Gamma - \frac{N_B}{L_n}D\Phi = 0, \tag{39}$$

$$\frac{1}{\varepsilon}W + N_{CT}(D^2 - a^2)\Theta + \left[\frac{(D^2 - a^2)}{L_e} - \frac{n}{\sigma}\right]\Gamma = 0, \tag{40}$$

where $D = \frac{d}{dz}$ *and* $a^2 = k_x^2 + k_y^2$ is the dimensionless wave number and the boundary conditions in view of normal mode are

$$W = D^2 W = \Gamma = \Theta = \Phi = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{41}$$

7. LINEAR STABILITY ANALYSIS

The eigen function $f_i(z)$ corresponding to the eigen value problem (37) - (41) are $f_j = \sin\,(\pi z)$.

Considering solutions $W,\ \Theta,\ \Gamma,\ \Phi$ of the form:

$$W = W_0 \sin(\pi z)\,, \Theta = \Theta_0 \sin(\pi z)\,, \Gamma = \Gamma_0 \sin(\pi z)\,, \Phi = \Phi_0 \sin\,(\pi z). \tag{42}$$

Substituting (42) into equations (37) - (40) and integrating each equation from$z = 0$ and $z = 1$, we obtain the following

$$\begin{bmatrix} \left(1 + nF + \frac{n}{\sigma V_a}\right)J^2 + Q\pi^2 + T_a\pi^2 & -a^2 R_a & -a^2\frac{R_S}{L_e} & a^2 R_N \\ \frac{1}{\varepsilon} & \frac{N_A}{L_n}J^2 & 0 & \frac{J^2}{L_n} + \frac{n}{\sigma} \\ -1 & J^2 + n & N_{TC}J^2 & 0 \\ -\frac{1}{\varepsilon} & N_{CT}J^2 & \frac{J^2}{L_e} + \frac{n}{\sigma} & 0 \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{43}$$

where $J^2 = \pi^2 + a^2$ is the total wave number.

The non-trivial solution of the above matrix equation (43) requires the determinant of the coefficients to vanish, which gives

$$R_a = \frac{1}{(J^2\sigma\varepsilon + n\varepsilon L_e - \sigma L_e N_{TC} J^2)} \left\{ \begin{array}{c} \frac{\varepsilon\left[\left(1+nF+\frac{n}{\sigma V_a}\right)J^2+Q\pi^2+T_a\pi^2\right]}{a^2}[(J^2+n)(\sigma J^2+nL_e)-\sigma L_e N_{CT} N_{TC} J^4] \\ +R_s\sigma[\varepsilon N_{CT} J^2-(J^2+n)] \\ -\frac{R_n\sigma}{(\sigma J^2+nL_n)}\left[\left((J^2+n)L_n+N_A\varepsilon J^2\right)(\sigma J^2+nL_e)-\sigma L_e N_{TC} J^4(L_n N_{CT}+N_A)\right] \end{array} \right\}, \quad (44)$$

where, $J^2 = \pi^2 + a^2$

8. THE STATIONARY CONVECTION

For the validity of principle of exchange of stabilities $n = 0$at the marginal stability, equation (44)reduce to

$$R_a^s = \frac{1}{(\varepsilon - L_e N_{TC})} \left\{ \begin{array}{c} \frac{\varepsilon[(\pi^2+a^2)^2+\pi^2(\pi^2+a^2)(Q+T_a)]}{a^2}[1-L_e N_{CT} N_{TC}] \\ -R_n[L_n+\varepsilon N_A - L_e N_{TC}(L_n N_{CT}+N_A)] \\ +R_s[\varepsilon N_{CT}-1] \end{array} \right\}. \quad (45)$$

The Darcy thermal Rayleigh number R_a^s given by equation (45)for stationary convection is a function of the dimensionless wave number a, thermo-nanofluid Lewis number L_n, thermosolutal Lewis number L_e, modified diffusivity ratio N_A, nanoparticle Rayleigh number R_N, solutal Rayleigh number R_s, Taylor number T_a, Chandrasekhar number Q, Soret parameter N_{CT}, Dufour parameter $N_{TC}$and medium porosity ε. Sinceelastico-viscous parameter F vanish with n so the Rivlin-Ericksen elastico-viscous nanofluid react like usual Newtonian nanofluid.

In the absence of the Dufour (N_{TC}) and Soret (N_{CT}) parameters equation (45), reduces to

$$R_a^s = \left[\frac{(\pi^2+a^2)[(\pi^2+a^2)+\pi^2 Q+\pi^2 T_a]}{a^2} - \frac{R_S}{\varepsilon} - R_N\left(\frac{L_n}{\varepsilon}+N_A\right)\right]. \quad (46)$$

The minimum value of R_a^s is obtained by putting $\frac{\partial R_a^s}{\partial a^2} = 0$, and which on simplification implies that

$$a_c{}^2 = \pi^2\sqrt{1+Q+T_a}, \quad (47)$$

where a_c is the critical wave number.

9. GRAPHICAL DISCUSSIONS

The variation of thermal Darcy-Rayleigh number with respect to wave number has been plotted using equation (45) for stationary case, The parameters for the nanofluid are the same as those given by Buongiorno [3]. They are as follows: $\varepsilon = 0.6, N_A = -5, R_N = -1, T_a = 50, Q = 100, R_S = 200, N_{CT} = 1, L_e = 20, N_{TC} = 0.2, L_n = 400$.

Figure 2 shows the thermal Darcy-Rayleigh number for stationary convection with respect to the non-dimensional wave number for three different thermo nanofluid Lewis number values $L_n = 300, 500, 700$. From the graph, we can see that the thermal Darcy-Rayleigh number increases with increasing values of the thermo-nanofluid Lewis number. Hence, the thermo-nanofluid Lewis number stabilizes the system.Figure 3 represents the variations of thermal Darcy-Rayleigh number with the wave number for three different values of the Taylor number $T_a = 100, 200, 300$. The thermal Darcy-Rayleigh number increases with an increase in Taylor number, which implies that the Taylor number stabilizes the system.

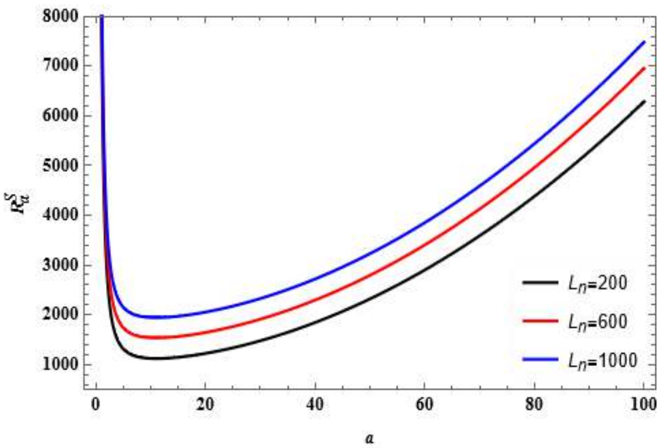

Figure 2: Variation of Thermal Darcy-Rayleigh number with Respect to the Wave Number for three different Values of the Thermo-Nanofluid Lewis Number.

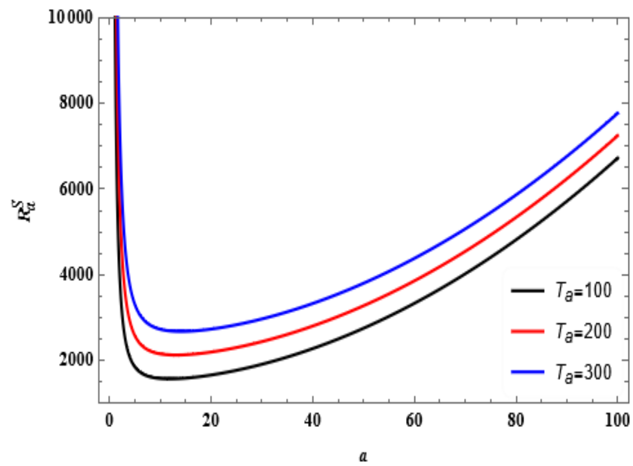

Figure 3: Variation of thermal Darcy-Rayleigh Number with respect to Wave Number for different Values of Taylor Number.

Figure 4 the variation of the thermal Darcy-Rayleigh number for stationary convection concerning the non-dimensional wave number for different values of the modified diffusivity ratio $N_A = -10, -5, -1$. The graph shows that with the increase in the modified diffusivity ratio, the thermal Darcy-Rayleigh number increases for the stationary convection which has stabilized the system. Figure 5 represents the variation of thermal Darcy-Rayleigh number a concerning different values of the medium porosity $\varepsilon = 0.3, 0.5, 0.7$. The thermal Darcy-Rayleigh number increases with an increase in medium porosity ε, which implies that medium porosity ε has a stabilizing effect on the system for stationary convection.

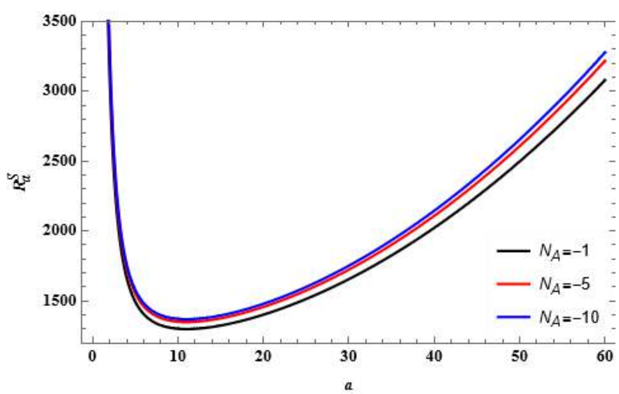

Figure 4: Variation of Thermal Darcy-Rayleigh Number with Respect to Wave number for different Values of Modified Diffusivity Ratio.

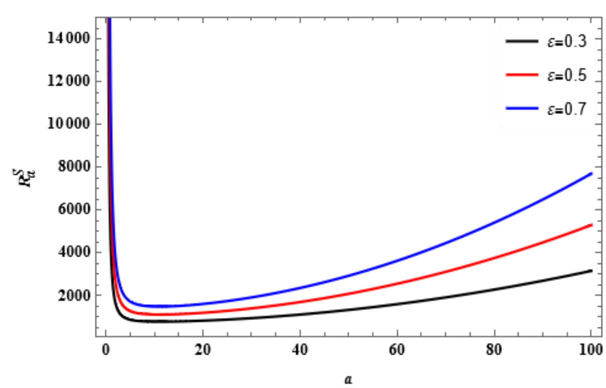

Figure 5: Variation of Thermal Darcy-Rayleigh Number with Respect to Wave Number for different Values of Medium Porosity.

Figure 6 shows the variation of the thermal Darcy-Rayleigh number concerning the non-dimensional wave number for three different values of the nanoparticles Rayleigh number $R_N = -1, -5, -10$. It is depicted from the graphs that the thermal Darcy-Rayleigh number decreases with the increase in nanoparticle Rayleigh number which causes destabilize the system. Figure7 shows the variations of the thermal Darcy-Rayleigh number with the wave number for three different values of the solutal Rayleigh number, $R_S = 200, 400, 600$ and it is observed that the thermal Darcy-Rayleigh number increases with the increase in solutal Rayleigh number so the solutal Rayleigh number stabilizes the system.

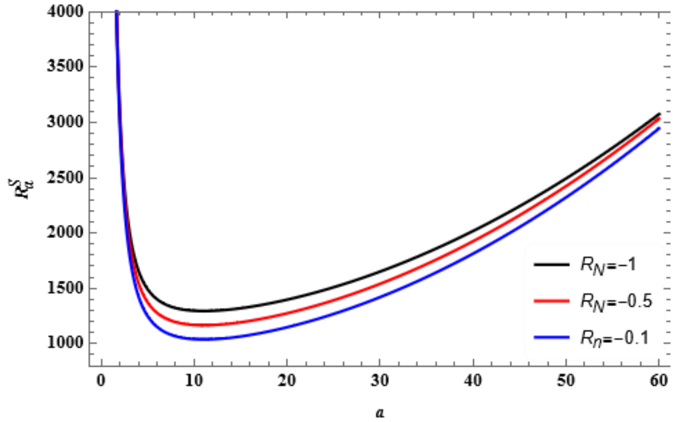

Figure 6: Variation of thermal Darcy-Rayleigh number with respect to wave number for different values of nanoparticle Rayleigh Number.

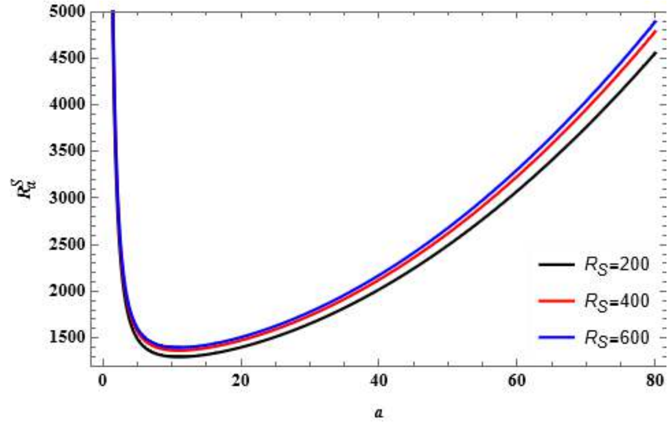

Figure 7: Variation of thermal Darcy-Rayleigh number with respect to wave number for different values of solutal Rayleigh Number.

Figure 8 the variations of thermal Darcy-Rayleigh number with the wave number a for three different values of the Chandrasekhar number namely $Q = 100, 200, 300$ which shows that the thermal Darcy-Rayleigh number increases with the increase in Chandrasekhar number. Thus, Chandrasekhar number stabilizes the system. Figure 9 the effect of the Dufour parameter N_{TC} on neutral curves is displayed. The variations of the thermal Darcy-Rayleigh number with respect to the wave number for three different values of the Dufour parameter, namely $N_{TC} = 0.1, 0.2, 0.3$ shows that the thermal Darcy-Rayleigh number increases with the increase in the Dufour parameter. Thus, the Dufour parameter stabilizes the system for stationary convection.

Figure 10 the variations of the thermal Darcy-Rayleigh number with respect to the wave number for three different values of the Soret parameter, which shows that the thermal Darcy-Rayleigh number increases with the increase in the Soret parameter. Thus, the Soret parameter stabilizes the system. Figure 11 the variations of thermal Darcy-Rayleigh number with the wave number for three different values of the thermo-solutal Lewis number, which shows that the thermal Darcy-Rayleigh numberincreases with the increase in thermo-solutal Lewis number. Thus, the thermo-solutal Lewis number stabilizes the system.

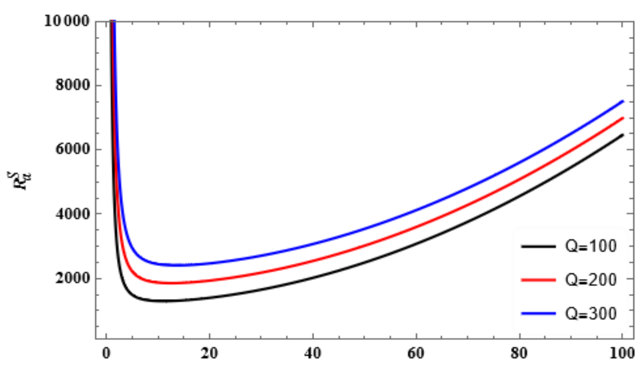

Figure 8: Variation of Thermal Darcy-Rayleigh Number with Respect to Wave number for different Values of

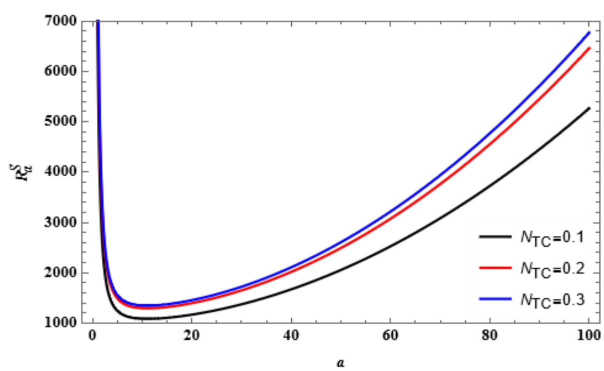

Figure 9: Variation of Thermal Darcy-Rayleigh number with Respect to wave number for Different

Chandrasekhar Number.

Values of with Dufour Parameter.

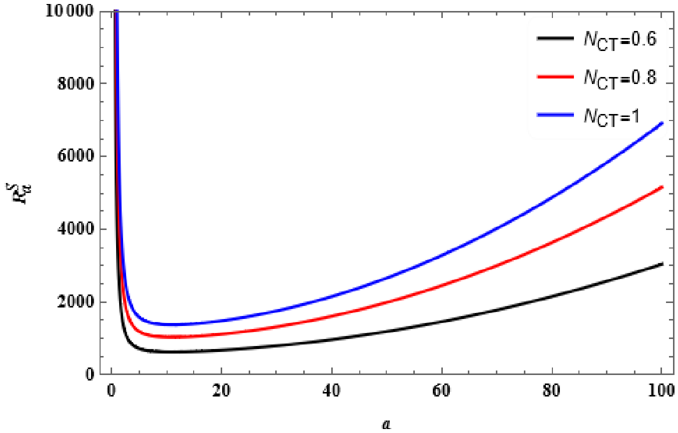

Figure 10: Variation of thermal Darcy-Rayleigh Number with Respect to wave Number for different Values of with Soret Parameter.

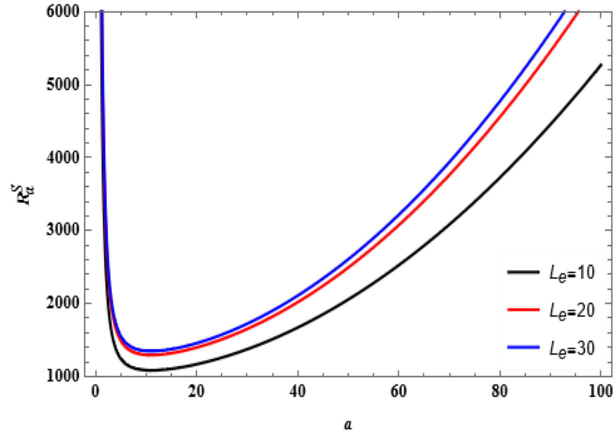

Figure 11: Variation of Thermal Darcy-Rayleigh Number with Respect to Wave Number for different values of Thermo-Solutal Lewis Number.

10. CONCLUSIONS

The initiation of thermal solute convection in elastic-viscous nanofluid in porous media in the presence of rotation and the magnetic field is studied by linear stability analysis. The main conclusions are:

- The thermal nanofluid Lewis number, Dufour parameter modified diffusion ratio, solutal Rayleigh number and Soret parameter have a stabilizing effect on steady-state convection.
- Nanoparticle Rayleigh number has destabilized the steady-state convection.
- The Chandrasekhar number, which describes the magnetic field, stabilizes steady-state convection.
- The Taylor number shows that rotation stabilizes the steady-state convection.
- Medium porosity and thermo-solutal Lewis number have stabilizing steady-state convection.

11. ACKNOWLEDGMENTS

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12. REFERENCES

1 *G. Veronis, "On finite amplitude instability in thermohaline convection" Journal of Marine Research 23 (1965) 1.*

2 *J. A. Eastman, S. Choi, S. Li, W. Yu & L.J. Thompson, "Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles" Applied Physics Letters 78 (2001) 718.*

3 *J. Buongiorno, "Convective transport in nanofluids", ASME Journal of Heat Transfer 128 (2006) 240.*

4 *G. Rana & V. Sharma, "Hydromagnetic thermosolutal instability of compressible Walters' (Model B') rotating fluid permeated with suspended particles in a porous medium" The International Journal of Multiphysics 5 (2011) 325.*

5 *G. C. Rana, R. C. Thakur & S. Kumar, "Thermosolutal convection in compressible Walters' (Model B') fluid permeated with suspended particles in a Brinkman porous medium" The Journal of Computational Multiphase Flows 4 (2012) 211.*

6 *G. C. Rana & H. S. Jamwal, "Hydromagnetic thermal instability of compressible Walters' (Model B') rotating fluid permeated with suspended particles in a porous medium" Studia Geotechnica et Mechanica 35 (2013) 75.*

7 *G. C. Rana, R. C. Thakur, & S. K. Kango, "On the onset of double-diffusive convection in a layer of nanofluid under rotation saturating a porous medium" Journal of Porous Media, 17 (2014) 657.*

8 *G. C. Rana & R. Chand, "Stability analysis of double-diffusive convection of Rivlin-Ericksen elastico-viscous nanofluid saturating a porous medium: a revised model" ForschungimIngenieurwesen 79 (2015) 87.*

9 *R. Chand & G. C. Rana, "Magneto convection in a layer of nanofluid with soret effect" acta mechanica et automatica 9 (2015) 63.*

10 *P.L. Sharma, Deepak, & A. Kumar, "Effects of rotation and magnetic field on thermosolutal convection in elastico viscous Walters' (Model B') nanofluid with porous medium" Stochastic Modeling and Applications 26 (2022) 21.*

11 *P. Lata, A. Kumar, Deepak & G. C. Rana, "Effect of magnetic field on thermosolutal convection in Jeffrey nanofluid with porous medium" Special Topics & Reviews in Porous Media: An International Journal 14 (2023) 17.*

12 *P. L. Sharma, M. Kapalta, A. Kumar, D. Bains, S. Gupta, & P. Thakur, "Electrohydro dynamics convection in dielectric rotating Oldroydian nanofluid in porous medium" Journal of the Nigerian Society of Physical Sciences 5 (2023) 1231.*

13 *S. Chandrasekhar, Hydrodynamic and hydromagnetic stability. Courier Corporation (2013).*

14 *A. V. Kuznetsov, & D. A. Nield, "The onset of double-diffusive nanofluid convection in a layer of a saturated porous medium" Transport in porous media, 85 (2010) 941.*

15 *D. A. Nield, & A. V. Kuznetsov, "Thermal instability in a porous medium layer saturated by a nanofluid" International Journal of Heat and Mass Transfer 52 (2009) 5796.*

16 *L.J. Sheu, "Thermal instability in a porous medium layer saturated with a viscoelastic nanofluid" Transport in Porous Media, 88 (2010) 461.*